Multi-attribute Decision Analysis

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Multi-attribute Decision Analysis Methods

- Weighted Linear Combination
- Analytic Hierarchy Process
- Ideal Point Methods

Weighted Linear Combination

- Weighted Linear Combination (WLC) is the most often used GIS-MADA method

- WLC model consists of two components:
  - Criterion weights: w_k
  - Value functions: $v(a_{ik})$

Weighted Linear Combination

Associates with the ith decision alternative (location) a set of criterion weights, w_1, w_2,…, w_n, and combines the weights with the criterion (attribute) values, a_{i1}, a_{i2}, …, a_{in}, (i = 1, 2, …, m) as follows:

$$V(A_i) = \sum_{k=1}^{n} w_k \, v(a_{ik})$$

overall value of the i-th alternative at location, s_i, defined by the (x_i, y_i) coordinates

value of the ith alternative with respect to the k-th attribute measured by means of the value function

- The alternative characterized by the highest value of V(Ai) is the most preferred one

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Proximity-Adjusted WLC

- Based on the idea of adjusting preferences according to the spatial relationship between alternatives, or an alternative and some reference locations

$$V(A_i^p) = \sum_{k=1}^{n} w_{ik}\, v(a_{ik})$$

proximity-adjusted weight,
assigned to the i-th alternative
with respect to the
K-th criterion

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Local WLC

overall value of the i-th alternative estimated locally (in the q-th neighbourhood)

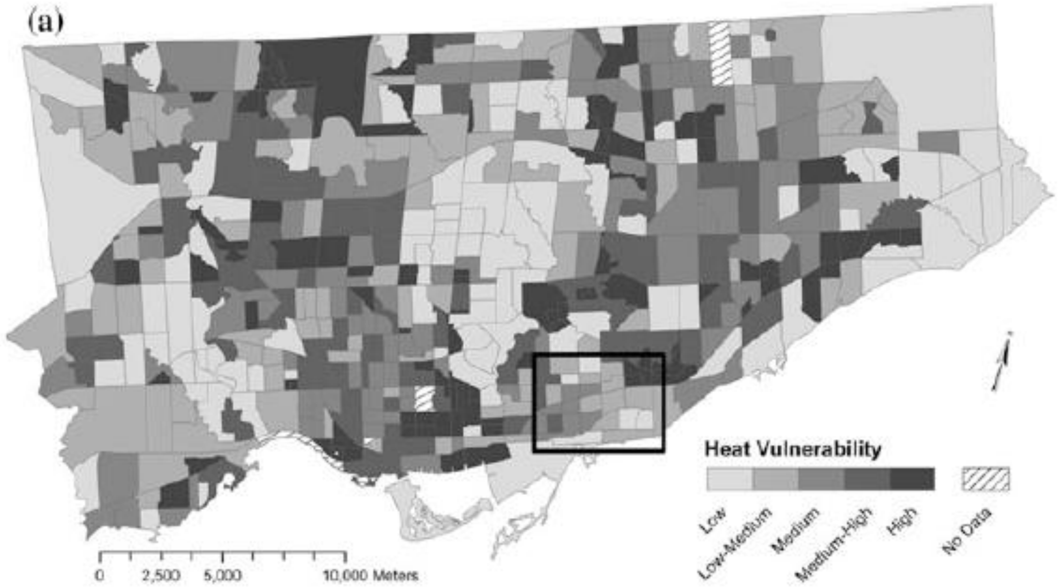
$$V(A_i^q) = \sum_{k=1}^{n} w_k^q \, v(a_{ik}^q)$$

local criterion weight

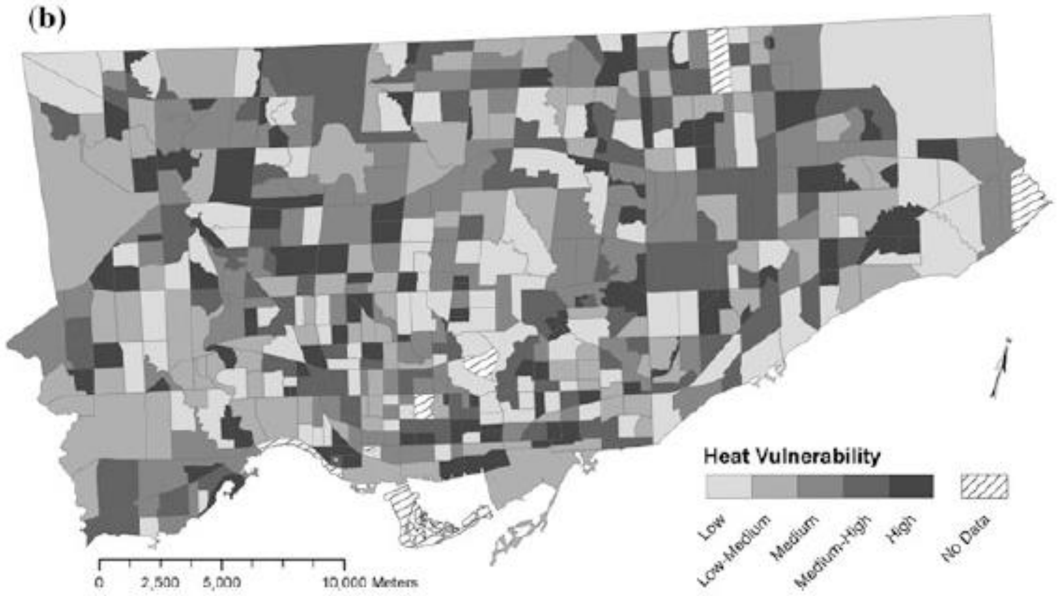
value of the k-th criterion measured by means of the local value function in the q-th neighbourhood

- The decision alternative with the highest value of $V(A_i^q)$ is the most preferred alternative in the q-th neighbourhood.

global vs. local WLC models



global WLC



local WLC

Heat vulnerability in Toronto modelled using: (a) global WLC, and (b) local WLC

Multiattribute Decision Analysis Methods

- Weighted Linear Combination
- **Analytic Hierarchy Process**
- Ideal Point Methods

Analytic Hierarchy Process

- One of the most comprehensive methods of multicriteria decision analysis
- Based on three principles:
  - decomposition
    - requires that a decision problem be decomposed into a hierarchy that captures the essential elements of the problem
  - comparative judgment
    - requires assessment of pairwise comparisons of the elements within a given level of the hierarchical structure, with respect to their parent in the next-higher level
  - synthesis of priorities
    - takes each of the derived ratio scale priorities in the various levels of the hierarchy and constructs a composite set of priorities for the elements at the lowest level of the hierarchy (that is, alternatives)

Analytic Hierarchy Process steps

AHP procedure involves three main steps:

1. developing the AHP hierarchy

2. assigning weights of importance to each element of the hierarchical structure
   - using the pairwise comparison method

3. constructing an overall priority rating

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Hierarchical Structure

four levels:

- goal
- objectives
- attributes
- alternatives

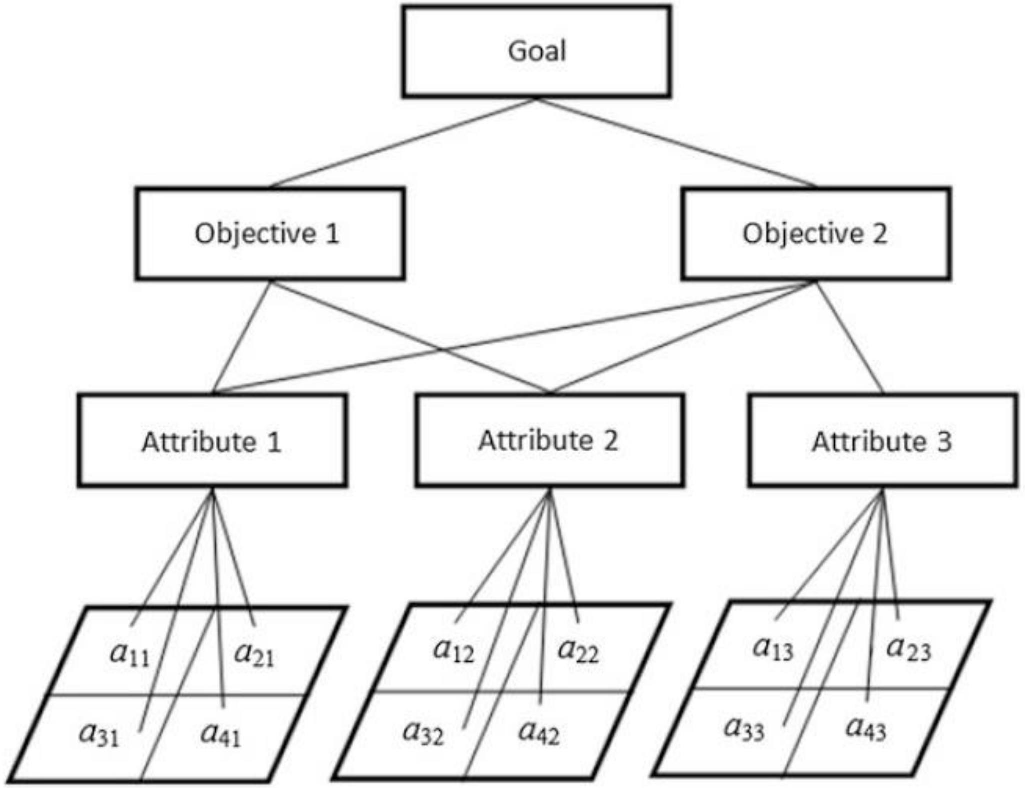


Fig. 2.1 Hierarchical structure of decision problem; a_{ik} is the value of the k-th attribute (criterion) associated with the i-th alternative (k = 1, 2, 3, and i = 1, 2, 3, 4)

Assigning weights of importance through Pairwise comparison

- Normalization of Matrix C entries:

$$c^*_{kp} = \frac{c_{kp}}{\sum_{k=1}^{n} c_{kp}}, \text{ for all } k = 1, 2, \ldots, n.$$

- then the weights are computed as follows

$$w_k = \frac{\sum_{p=1}^{n} c^*_{kp}}{n}, \text{ for all } k = 1, 2, \ \ldots, \ n.$$

Pairwise Comparison weighting example

Pairwise comparisons of:

(a) Objectives with respect to goal

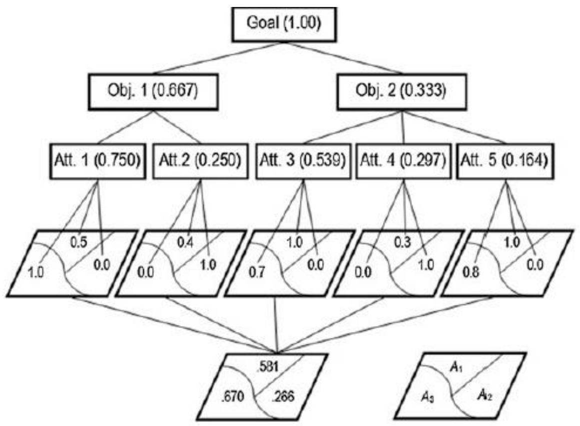
(b) attributes with respect to objective 1

(c) Attributes with respect to objective 2

(a)			
Goal			
Objectives	Obj.1	Obj. 2	w_l
Obj.1	1	2	0.667
Obj.2	0.5	1	0.333
Sum	1.5	3.0	1.000
$CR = 0.00$			

(b)			
Objective 1			
Attributes	Att.1	Att.2	$w_{k(1)}$
Att.1	1	3	0.750
Att.2	0.33	1	0.250
Sum	1.33	4.00	1.000
$CR = 0.00$			

(c)				
Objective 2				
Attributes	Att.3	Att.4	Att.5	$w_{k(2)}$
Att.3	1	2	3	0.539
Att.4	0.5	1	2	0.297
Att.5	0.33	0.5	1	0.164
Sum	1.83	3.50	6.00	1.000
$CR = 0.01$				



Analytic Hierarchy Process

- Global Method

weights assigned to the k-th attribute associated with the l-th objective

$$V(A_i)=\sum_{k=1}^{n} w_l w_{k(l)}\, v(a_{ik})$$

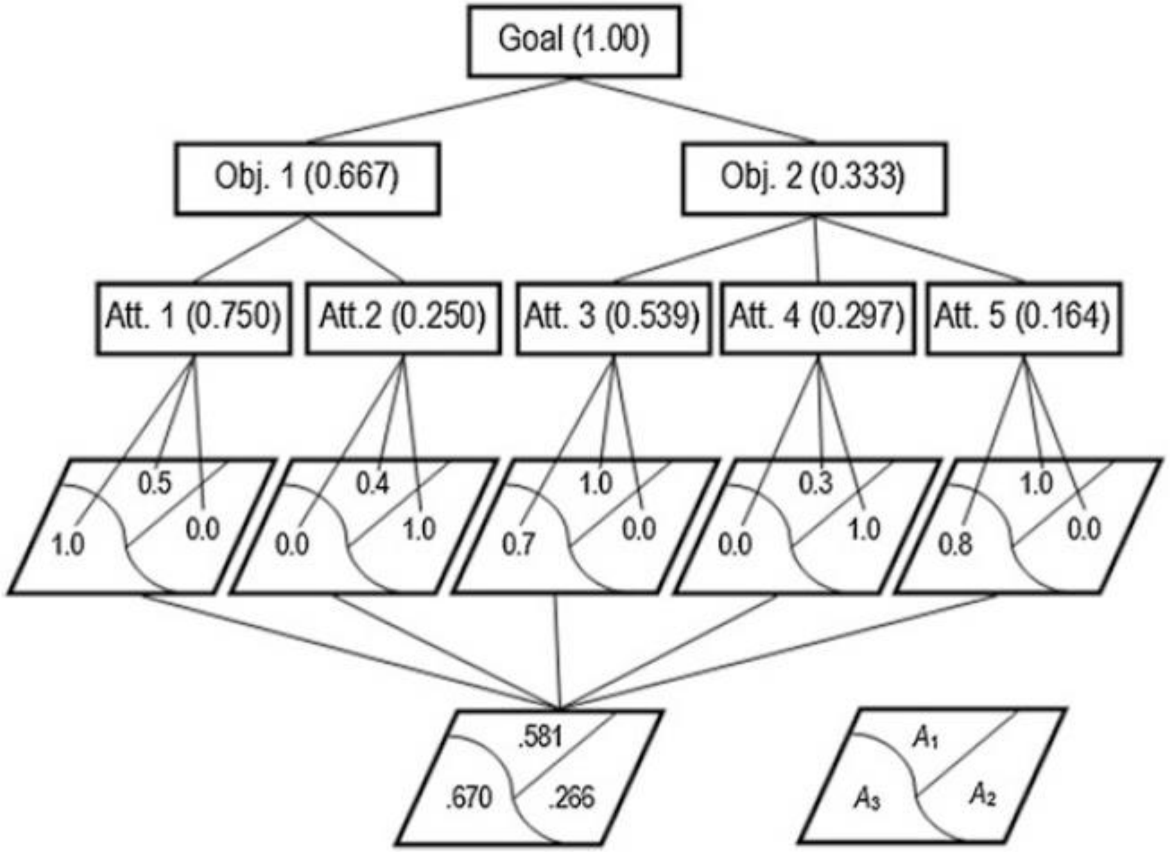
weights associated with the l-th objective (l = 1, 2,…, p)

GIS-based AHP model example

A1, A2, A3: Alternatives

Obj: Objective

Att: Attribute



evaluating three parcels of land (A1, A2, and A3)

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Goal			
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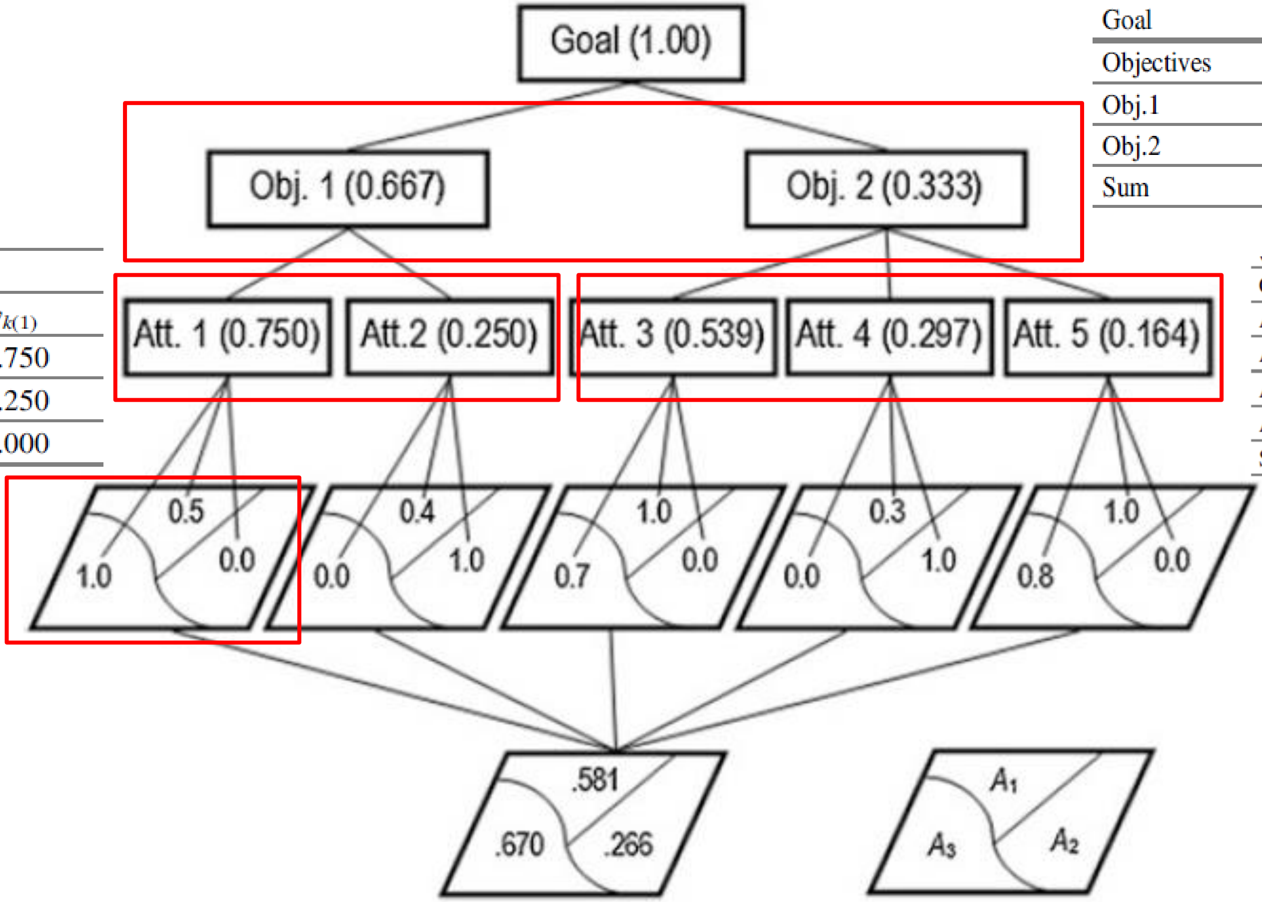

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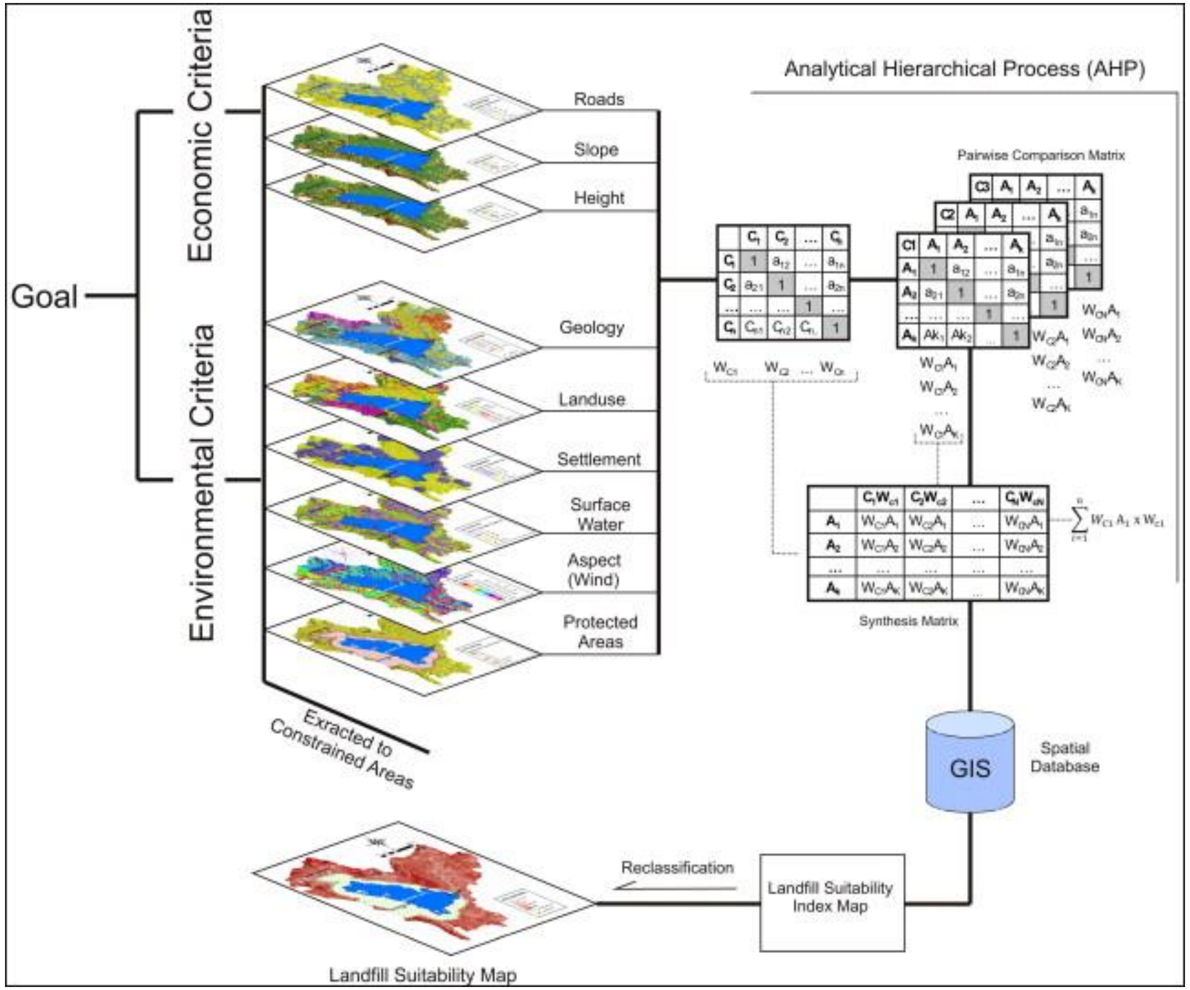
Sum	1.83	3.50	6.00	1.000

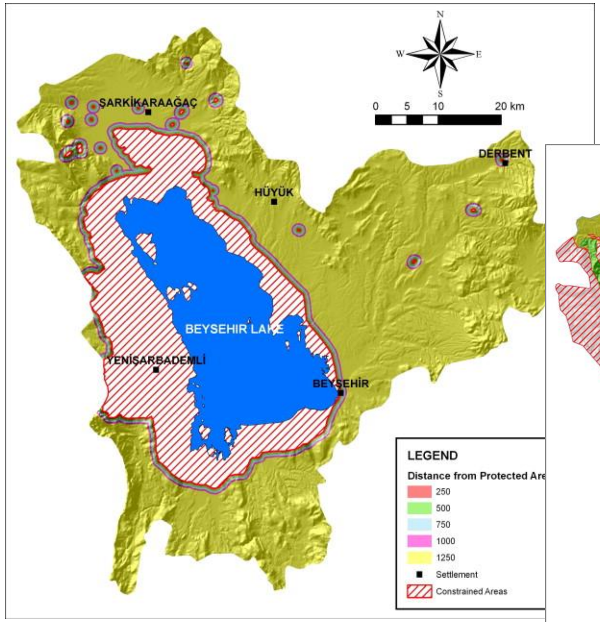
standardized attribute values for each parcel of land

GIS-based AHP example for GIS for landfill site selection

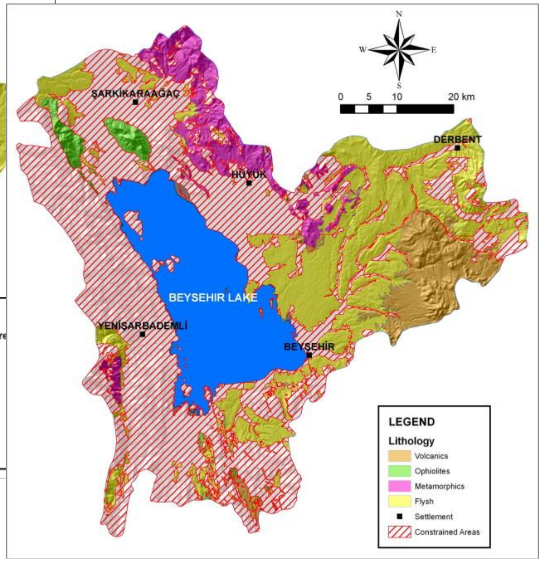


Source: Şener, Ş., Şener, E., Nas, B., & Karagüzel, R. (2010). Combining AHP with GIS for landfill site selection: a case study in the Lake Beyşehir catchment area (Konya, Turkey). Waste management, 30(11), 2037-2046.

GIS-based AHP example for GIS for landfill site selection



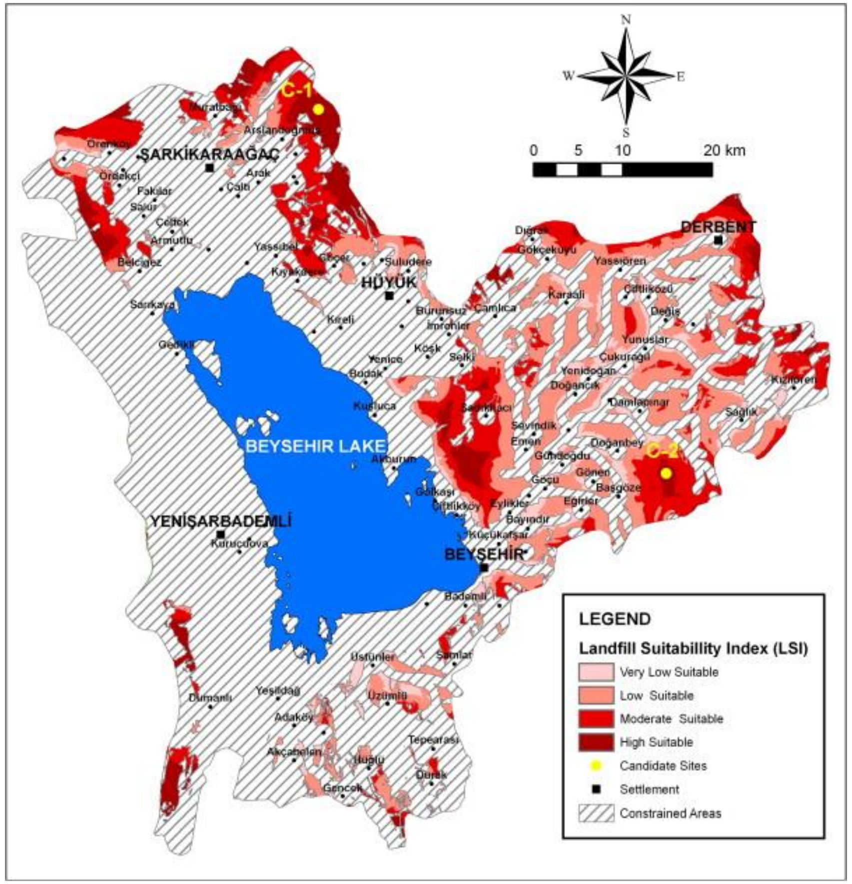
Map of the distance
from protected areas



Geology map

Landuse Map
Slope/height Map
etc.
……………

GIS-based AHP example for GIS for landfill site selection



Landfill suitability map

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Multiattribute Decision Analysis Methods

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- **Ideal Point Methods**

Ideal Point Methods

- Based on evaluating decision alternatives with reference to some specific target or goal
- Ordering a set of decision alternatives on the basis of their separations from some ideal/reference point

Reference Points and Separation Measures

- Reference point: any significant target or goal against which the decision alternatives are evaluated

- This hypothetical alternative is often defined in terms of the **positive ideal** (utopia) point, or **negative ideal** (or anti-ideal or nadir) point

- Positive and negative ideal points are determined as the best and worst-possible value achievable by any alternative, respectively:

positive ideal alternative $A^+ = t_1^+, t_2^+$ where $t_k^+ = \max_i \{v(a_{ik})\}$

negative ideal alternative $A^- = t_1^-, t_2^-, \ldots, t_n^-$ where $t_k^- = \min_i \{v(a_{ik})\}$

Reference Points and Separation Measures

Based on the definition of value function $v(a_{ik})$:

$$A^- = (0, 0, \ldots, 0)$$

$$A^+ = (1,1,\ldots,1)$$

separation of the i-th decision alternative from a reference point can be defined by means of the L_p distance metric as follows :

reference value for the kth criterion, e.g., t_k^+ / t_k^-

$$L_p(A_i) = \left[\sum_{k=1}^{n} (w_k |t_k - v(a_{ik})|)^p \right]^{\frac{1}{p}}$$

power parameter ranging from 1 to ∞

Distance Metric

If p=1 → $$L_1(A_i) = \sum_{k=1}^{n} w_k |t_k - v(a_{ik})|$$ rectangular distance (Manhattan metric)

If p=2 → $$L_2(A_i) = \sqrt{\sum_{k=1}^{n} (w_k |t_k - v(a_{ik})|)^2}$$ straight-line distance

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Ideal Point Models

Two versions of the ideal point model:

positive ideal models

$$A^{+} = (1,1, \ldots,1)$$

$$L_p^{+}(A_i) = \left[\sum_{k=1}^{n} (w_k |1 - v(a_{ik})|)^p \right]^{\frac{1}{p}}$$

The best alternative is that one which **minimizes** the value of $L_p^{+}(A_i)$

negative ideal models

$$A^{-} = (0, 0, \ldots, 0)$$

$$L_p^{-}(A_i) = \left[\sum_{k=1}^{n} (w_k v(a_{ik}))^p \right]^{\frac{1}{p}}$$

The best alternative is that one which **maximizes** the value of $L_p^{-}(A_i)$

Ideal Point versus WLC

If p=1 →

$$L_p^+(A_i) = 1 - V(A_i)$$

$$\boxed{L_p^-(A_i) = V(A_i)}$$

the negative ideal model for p = 1 is completely equivalent to WLC

Example

Simple problem of evaluating 16 location (rasters) based on two criteria:

The criteria are to be **maximized**

criterion weights:

w_1=0.6

w_2=0.4

a_{i1}

5.5	5.0	3.0	1.0
3.5	6.5	10.5	9.5
1.0	0.0	8.5	6.0
3.5	1.5	12.0	10.5

a_{i2}

0.0	1.0	2.0	3.0
1.0	1.4	2.2	3.2
2.0	2.0	2.8	3.6
3.0	3.2	3.6	4.2

Positive ideal model

overall values of alternatives using the ideal point $A^{+} = (1.0,\ 1.0)$

.xxx best alternative

(a)

.275	.345	.340	.336
.270	.458	.735	.780
.240	.210	.692	.643
.461	.380	.943	.925

$L_1^{+}(A_i)$ model

(b)

.275	.268	.242	.290
.199	.351	.565	.564
.197	.210	.502	.456
.335	.314	.691	.660

$L_2^{+}(A_i)$ model

(c)

.275	.250	.192	.286
.175	.325	.525	.476
.190	.210	.425	.351
.286	.305	.600	.528

$L_\infty^{+}(A_i)$ model

Negative ideal model

overall values of alternatives using the ideal point $A^{-} = (0.0, 0,0)$

.xxx best alternative

(a)

.725	.655	.660	.664
.730	.542	.265	.220
.760	**.790**	.308	.357
.539	.620	.057	.075

$L_1^{-}(A_i)$ model

(b)

.515	.464	.496	.562
.523	.383	.205	.157
.587	**.630**	.220	.305
.440	.534	.057	.075

$L_2^{-}(A_i)$ model

(c)

.405	.358	.450	.550
.427	.291	.190	.126
.550	**.600**	.176	.300
.425	.525	.057	.075

$L_\infty^{-}(A_i)$ model

Spatially explicit ideal point model

value of the i-th alternative at the x, y location for the k-th criterion

$$L_p^+(A_{i(x,y)}) = \left[\sum_{k=1}^{n} \left(w_{i(x,y)k} \left| 1 - v(a_{i(x,y)k}) \right| \right)^p \right]^{\frac{1}{p}}$$

weight associated with the kth criterion and the i-th alternative at the x, y location