GIS-MCDA

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Course Content

- Elements of Multi-Criteria Decision Analysis (MCDA)

- MCDA basis concepts:
  - Value scaling
  - Criterion Weighting
  - Combination Rules

Multi-criteria decision problem

- “a multi-criteria decision problem involves a set of **alternatives** that are evaluated on the basis of conflicting and incommensurate **criteria** according to the **decision maker**’s preferences.”

- Three main elements:
  - decision maker(s)
  - Alternatives
  - criteria

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Elements of MCDA

- Decision Makers
- Criteria
- Decision Alternatives

Elements of MCDA

- **Decision Makers**
- Criteria
- Decision Alternatives

Decision Makers

- Decision maker: an entity with the responsibility to make decisions
  - Individual (e.g., searching for a house or an apartment)
  - A group of individuals (e.g., selecting a suitable site for housing development)
  - An organization (e.g., allocating resources for housing development)

- Distinction between individual and multiple decision makers depends on the **consistency** of the group's goals, preferences, and beliefs rather than on the number of individuals actually involved

- Many spatial decisions are made by groups (multiple decision makers) rather than an individual decision maker

Elements of MCDA

- Decision Makers
- **Criteria**
- Decision Alternatives

Criteria

- Decision alternatives are evaluated on the basis of a set of criteria

- Criteria includes:
  - Objectives
  - Attributes

Objectives

- Objective: a statement about the desired state of a system under consideration
  - Example: a spatial pattern of accessibility to primary schools
- Objective indicates the directions of improvement of one or more attributes
- Either 'the more of the attribute, the better' or 'the less of the attribute, the better'
  - This implies a **maximization** or **minimization** of an objective function

Attributes

- Attribute: a property of an element of a real-world geographic system (e.g., transportation system, location-allocation system)

- Example: For the objective of maximizing physical accessibility to schools, the attributes such as total traveling distance, time, cost, or any other measure of spatial proximity

Hierarchical Structure

- The relationships between objectives and attributes have a hierarchical structure

Hierarchical Structure

four levels:

- goal
- objectives
- Attributes
- alternatives

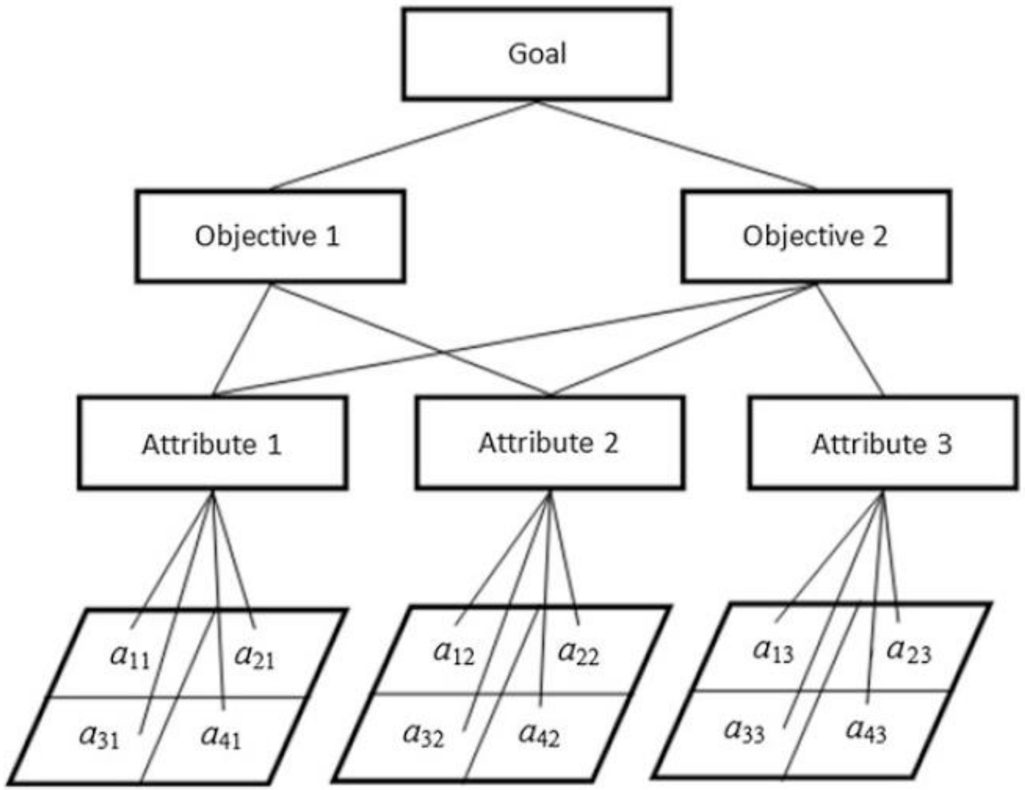

Fig. 2.1 Hierarchical structure of decision problem; a_{ik} is the value of the k-th attribute (criterion) associated with the i-th alternative (k = 1, 2, 3, and i = 1, 2, 3, 4)

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Elements of MCDA

- Decision Makers
- Criteria
- **Decision Alternatives**

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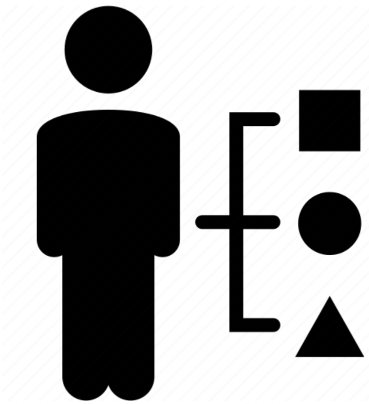
Decision Alternatives



- Decision alternatives: alternative courses of action among which the decision maker (agent) must choose

- A geographic decision alternative consists of at least two elements:
  - action (what to do?)
  - location (where to do it?)

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Decision variables

- An alternative is completely specified by defining the values of the decision variables

- Decision variables can be classified into three categories:
  - binary
    - yes/no decision
  - discrete
    - Example: number of patrons at a shopping mall
  - Continuous
    - Example: facility size

Feasible Alternatives

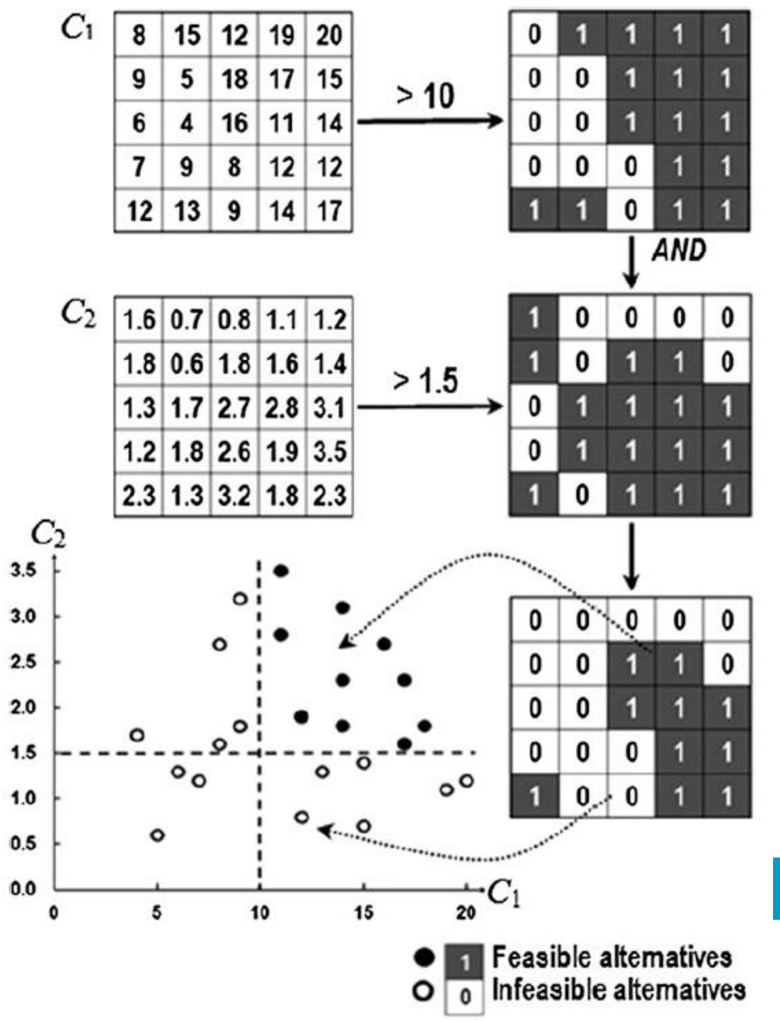
- Constraints represent restrictions imposed on the decision variables (alternatives)

- They divide decision alternatives into two categories:
  - acceptable (feasible)
  - unacceptable (infeasible)

- An alternative is feasible if it satisfies all constraints

Feasible and infeasible decision alternatives for two criteria

Feasible and infeasible decision alternatives for two criteria: C1 and C2, and constrains C1 > 10 and C2 > 1.5



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Decision Matrix

Elements of MCDA

	Criterion/attribute, C_k					Coordinates	
Alternative, A_i	C_1	C_2	C_3	…	C_n	X	Y
A_1	a_{11}	a_{12}	a_{13}	…	a_{1n}	x_1	y_1
A_2	a_{21}	a_{22}	a_{23}	…	a_{2n}	x_2	y_2
A_3	a_{31}	a_{32}	a_{33}	…	a_{3n}	x_3	y_3
…	…	…	…	…	…	…	…
A_m	a_{m1}	a_{m2}	a_{m3}	…	a_{mn}	x_m	y_m
Weight, w_k	w_1	w_2	w_3	…	w_n	w_{ik}	

The elements of MCDA can be organized in a tabular format.

MCDA basic concepts

- Value scaling
- Criterion Weighting
- Combination Rules

MCDA basic concepts

- **Value scaling**
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Value Scaling

- Requirement for transforming the evaluation criteria to comparable

Units

- The procedures for transforming raw data to comparable units are referred to as the **value scaling** or **standardization** methods.

- **Score range procedure** is the most popular GIS-based method for standardizing evaluation criteria

Value Function

- Mathematical representation of human judgment
- Worth or desirability of that alternative with respect to that criterion

Value Function

$$v(a_{ik}) = \left(\frac{\max_i\{a_{ik}\} - a_{ik}}{r_k} \right)^{\rho},$$ for the k-th criterion to be minimized;

$$v(a_{ik}) = \left(\frac{a_{ik} - \min_i\{a_{ik}\}}{r_k} \right)^{\rho},$$ for the k-th criterion to be maximized;

a_{ik} : level of the k-th criterion (k = 1, 2, …, n) for the i-th alternative (i = 1, 2, …, m)

$\min_i a_{ik}$ minimum criterion values for the k-th criterion

$\max_i a_{ik}$ maximum criterion values for the k-th criterion

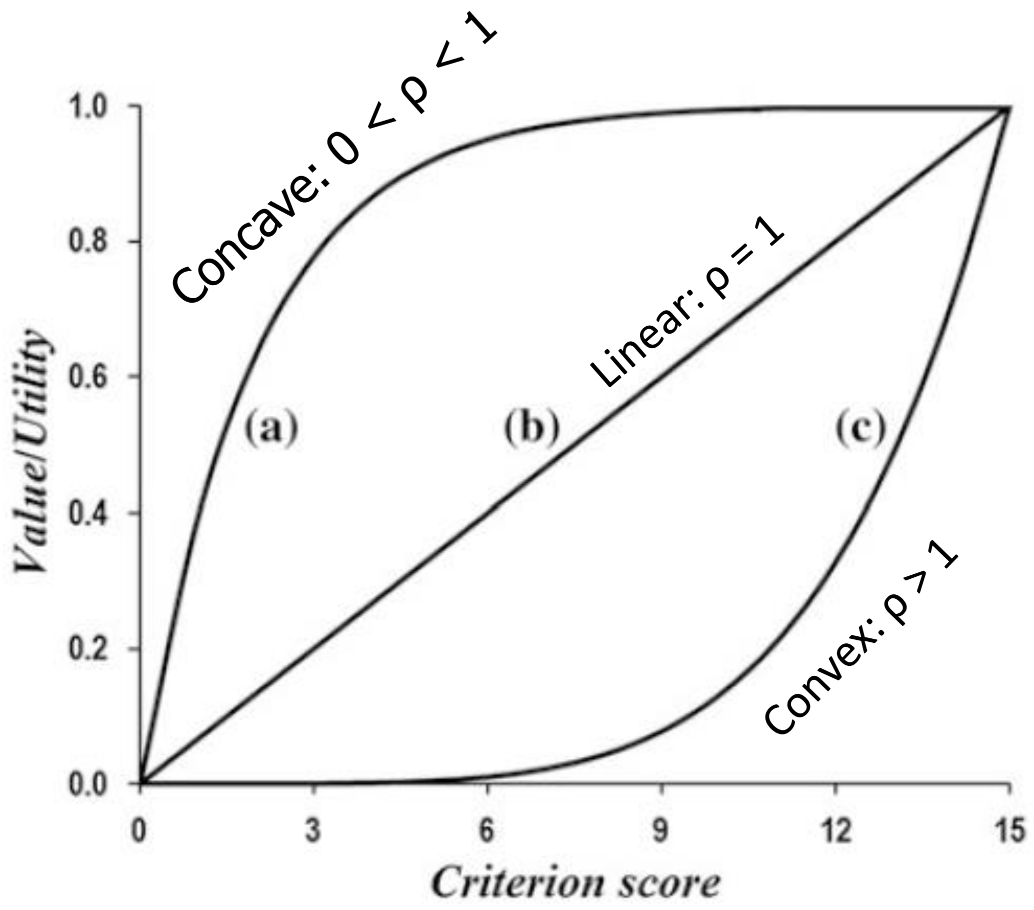
$$r_k = \max_i\{a_{ik}\} - \min_i\{a_{ik}\}$$

range of the k-th criterion

Value Function

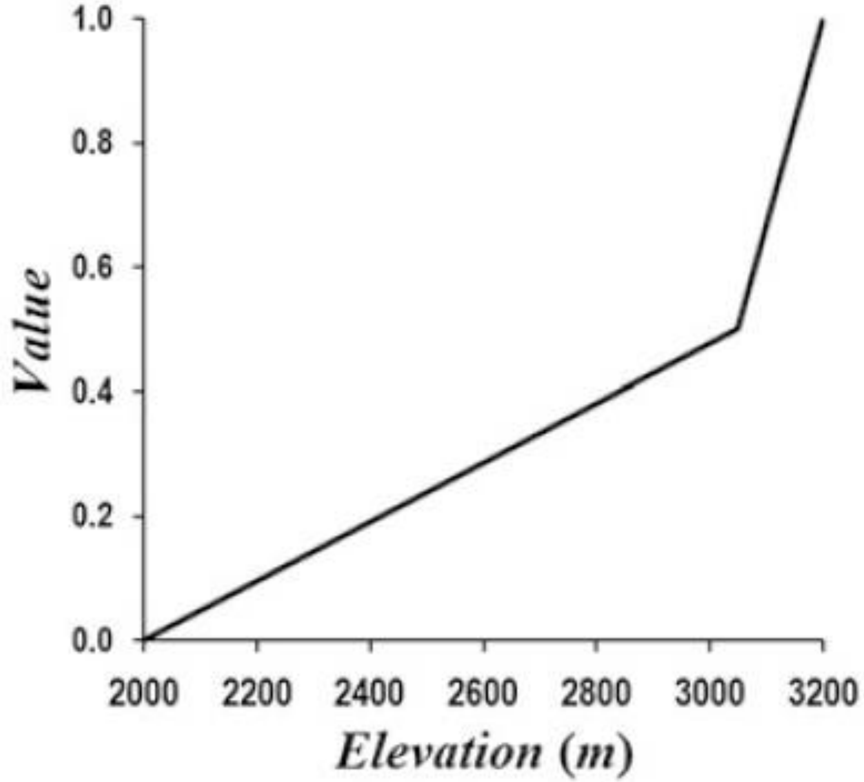
- standardized score values $v(a_{ik})$ range from 0 to 1:
  - 0: the value of the least-desirable outcome
  - 1: the most-desirable score

Value Function



Piecewise linear form value function

- In real-world applications of GIS-MCDA, the value function is often approximated by a piecewise linear form



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Local value function

- global value function does not take into account spatial heterogeneity of the preferences that are represented by the relationship between the criterion score, a_{ik}, and the worth of that score, $v(a_{ik})$

- spatial variation of the value function can be operationalized by the concept of the local range:

$$r_k^q = \max_{iq}\{a_{ik}^q\} - \min_{iq}\{a_{ik}^q\},$$

$\min_{iq}\{a_{ik}^q\}$
$\max_{iq}\{a_{ik}^q\}$
minimum and maximum values of the k-th criterion in the q-th subset (q = 1, 2, …, g) of the locations, i = 1, 2, …, m; m > q, respectively.

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Local value function

$$v(a_{ik}^{q}) = \left(\frac{\max_{i,q}\{a_{ik}^{q}\} - a_{ik}^{q}}{r_k^{q}} \right)^{\rho_{(q)}},$$

for the k-th criterion to be minimized;

$$v(a_{ik}^{q}) = \left(\frac{a_{ik}^{q} - \min_{i,q}\{a_{ik}^{q}\}}{r_k^{q}} \right)^{\rho_{(q)}},$$

for the k-th criterion to be maximized;

MCDA basic concepts

- Value scaling
- **Criterion Weighting**
- Combination Rules

Criterion Weighting

- Weight: a value assigned to an evaluation criterion that indicates its **importance** relative to the other criteria under consideration.

- Weighting methods can be classified into two categories of:
  - Global methods
    - based on the assumption of spatial homogeneity of preferences
  - Local Methods
    - Taking into account spatial heterogeneity of preferences

General Properties of criterion weights

- Criterion weights, w_1, w_2, …, w_k, …,w_n should follow:

$$0 \leq w_k \leq 1 \text{ and } \sum_{k=1}^{n} w_k = 1$$

- Weights must be ratio scaled:
  - If criterion C1 is twice as ‘important’ as C2, then w1 = 2w2; that is, w1 = 0.667 and w2 = 0.333.

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Global Criteria Weighting

- Ranking Method
- Rating Method
- Pairwise comparison
- Entropy-Based Criterion Weights

Global Criteria Weighting

- **Ranking Method**
- Rating Method
- Pairwise comparison
- Entropy-Based Criterion Weights

Ranking Method

- Rank the criteria in the order of the decision maker's preference

Steps:

- Straight ranking (the most important = 1, second important = 2, etc.)
- Estimation of k-th criterion weight w_k:

$$w_k = \frac{n - p_k + 1}{\sum_{k=1}^{n}(n - p_k + 1)}$$

n: number of criteria under consideration

p_k: rank position of the criterion

Global Criteria Weighting

- Ranking Method
- **Rating Method**
- Pairwise comparison
- Entropy-Based Criterion Weights

Rating Method

- Decision makers estimate weights on the basis of a predetermined scale; for example, a scale of 0 to 100
- Given the scale, a score of 100 is assigned to the most important criterion.
- Proportionately smaller weights are then given to criteria lower in the order.
- The procedure is continued until a score is assigned to the least important criterion
- Finally, the weights are normalized by dividing each of the weights by the sum total.

Global Criteria Weighting

- Ranking Method
- Rating Method
- **Pairwise comparison**
- Entropy-Based Criterion Weights

Pairwise comparison

- Employs an underlying scale with values from 1 to 9 to rate the preferences with respect to a pair of criteria

- Pairwise comparisons are organized into a matrix: $C = [c_{kp}]_{n \times n}$

  c_{kp}: pairwise comparison rating for the k-th and p-th criteria

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Approximating the values of criterion weights

Averaging over normalized columns

- Normalization of Matrix C entries:

$$c^*_{kp} = \frac{c_{kp}}{\sum_{k=1}^{n} c_{kp}}, \text{ for all } k = 1, 2, \ldots, n.$$

- Then the weights are computed as follows

$$w_k = \frac{\sum_{p=1}^{n} c^*_{kp}}{n}, \text{ for all } k = 1, 2, \ \ldots, \ n.$$

Pairwise comparison
Example

	C1	C2	C3	C4	C5	Criteria Weight
C1	1	1/3	1/5	1/9	1/3	0.042
C2	3	1	1	1/5	1	0.122
C3	5	1	1	1/5	3	0.180
C4	9	5	5	1	5	0.552
C5	3	1	1/3	1/5	1	0.104

Global Criteria Weighting

- Ranking Method
- Rating Method
- Pairwise comparison
- **Entropy-Based Criterion Weights**

Entropy-Based Criterion Weights

- Unlike the ranking, rating, and pairwise comparison methods, the entropy-based criterion weighting approach does not require the decision making agents to specify their preferences with respect to the evaluation criteria.

- Entropy-Based Criterion Weights method is based on the concept of **information entropy**.

- Entropy: a measure of the expected information content of a massage

Entropy-Based Criterion Weights

Decision Matrix

	Criterion/attribute, C_k					Coordinates	
Alternative, A_i	C_1	C_2	C_3	…	C_n	X	Y
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A_3	a_{31}	a_{32}	a_{33}	…	a_{3n}	x_3	y_3
…	…	…	…	…	…	…	…
A_m	a_{m1}	a_{m2}	a_{m3}	…	a_{mn}	x_m	y_m
Weight, w_k	w_1	w_2	w_3	…	w_n	w_{ik}	

Entropy: $$E_k = -\frac{\sum_{i=1}^{m} p_{ik} \ln(p_{ik})}{\ln(m)}$$

$$p_{ik} = a_{ik} / \sum_{i=1}^{m} a_{ik}$$

a_{ik} : value of the k-th attribute for the i-th alternatives

$$w_{E_k} = \frac{b_k}{\sum_{k=1}^{n} b_k}$$

$$b_k = 1 - E_k$$

degree of diversity of the information contained in a set of criterion values

Entropy-Based Criterion Weights

- The entropy-based criterion weights can be combined with weights, w_k, obtained using one of the other methods discussed:

$$w_{E_k}^* = \frac{w_{E_k} w_k}{\sum_{k=1}^{n} w_{E_k} w_k}$$

- The values of the entropy-based criterion weights, w_{Ek} and w_{Ek}^* range from 0 to 1

Criterion Weighting

- Global Criteria Weighting
- **Local Criteria Weighting (Spatially Explicit Methods)**

Local Criteria Weighting (Spatially Explicit Methods)

- Proximity-Adjusted Criterion Weights
- Range-Based Local Criterion Weights
- Entropy-Based Local Criterion Weights

Local Criteria Weighting (Spatially Explicit Methods)

- **Proximity-Adjusted Criterion Weights**
- Range-Based Local Criterion Weights
- Entropy-Based Local Criterion Weights

Proximity-Adjusted Criterion Weights

- Adjusting preferences according to the spatial relationship between alternatives
- This method explicitly acknowledges the concept of spatial heterogeneity of preferences
- Proximity-adjusted criterion weights by introducing a **reference or benchmark location**

Proximity-Adjusted Criterion Weights

- The weights should reflect both:
  - relative importance of the criterion
    - assessed in terms of the global criterion weight
  - spatial position of a decision alternative with respect to a reference location
    - assessed in terms of a distance decay function; the closer a given alternative is situated to a reference location, the higher the value of the criterion weight should be.

Proximity-Adjusted Criterion Weights

global criterion weight

$$w_{ik} = w_k \frac{d_{ij}^s}{\frac{1}{m}\sum_{i=1}^{m} d_{ij}^s}$$

proximity-adjusted criterion weight assigned to the i-th alternative with respect to the k-th criterion

standardized distance for a pair of i and j locations

$$d_{ij}^s = \frac{\min_i \{d_{ij}\}}{d_{ij}}$$

distance between the i-th alternative and the j-th reference location

Local Criteria Weighting (Spatially Explicit Methods)

- Proximity-Adjusted Criterion Weights
- **Range-Based Local Criterion Weights**
- Entropy-Based Local Criterion Weights

Range-Based Local Criterion Weights

Other things being equal, the greater the range of values for the k-th criterion, the greater the weight, w_k, should be assigned to that criterion.

Local criterion weight

$$w_k^q = \frac{\frac{w_k r_k^q}{r_k}}{\sum_{k=1}^{n} \frac{w_k r_k^q}{r_k}}$$

r_k^q → Local range (for q-th neighbourhood)

r_k → global range

$$0 \le w_k^q \le 1, \quad \text{and} \sum_{k=1}^{n} w_k^q = 1$$

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Local Criteria Weighting (Spatially Explicit Methods)

- Proximity-Adjusted Criterion Weights
- Range-Based Local Criterion Weights
- **Entropy-Based Local Criterion Weights**

Entropy-Based Local Criterion Weights

$$w_{E_k}^q = \frac{1 - E_k^q}{\sum_{k=1}^{n} \left(1 - E_k^q\right)}, \quad 0 \leq w_{E_k}^q \leq 1, \quad \text{and} \quad \sum_{k=1}^{n} w_{E_k}^q = 1$$

$$E_k^q = -\frac{\sum_{i \in q} p_{ik}^q \ln(p_{ik}^q)}{\ln(|q|)}$$

$|q|$: number of decision alternatives located in the q-th neighbourhood

$$p_{ik}^q = a_{ik}^q \Big/ \sum_{i \in q} a_{ik}^q$$

a_{ik}^q: value of the k-th attribute for the i-th alternative located in the q-th neighbourhood

MCDA basic concepts

- Value scaling
- Criterion Weighting
- **Combination Rules**

Combination Rules

- Combination rule (decision rule) integrates the data and information about alternatives (**criterion maps**) and decision maker's preferences (**criterion weights**) into an **overall assessment of the alternatives**.

- Decision rules can be classified into four groups of:
  - Compensatory versus non-compensatory
  - **multiattribute versus multiobjective**
  - discrete versus continues methods
  - spatially implicit versus spatially explicit MCDA

Multiattribute and Multiobjective Methods

- Multicriteria decision rules can be broadly categorized into two groups:
  - **Multiattribute decision analysis (MADA)**
    - involve a predetermined, limited number of alternatives
    - outcome-oriented evaluation and choice process
  - **Multiobjective decision analysis (MODA)**
    - process-oriented design and search
    - make a distinction between the concept of decision variables and decision criteria

Multiattribute vs. Multiobjective Methods

	Multiattribute decision analysis (MADA)	Multiobjective decision analysis (MODA)
Examples of multicriteria methods	Weighted linear combination Analytic hierarchy/network process Outranking methods Ideal point methods	Linear/integer programming Goal programming Compromise programming Heuristics/metaheuristics
Examples of spatial decision problems	Site selection Land use/suitability Vulnerability analysis Environmental impact assessment	Site search Location-allocation Transportation problem Shortest path problem Districting

Discrete and Continuous Methods

- Overlaps with the multiattribute/ multiobjective dichotomy
- Example: Site Selection (discrete) versus site search (continuous) problems
- **Site Selection**: identify the best site for some activity given the set of potential (feasible) sites
    - All the characteristics (such as location, size, and relevant attributes) of the candidate sites are known
    - The problem is to rate or rank the alternative sites based on their characteristics so that the best site (or a set of sites) can be identified
- **Site Search** Analysis: No pre-determined set of candidate sites
    - The characteristics of the sites (i.e., their boundaries) have to be defined by solving the problem
    - The aim of the site search analysis is to explicitly identify the boundary of the best site(s)

Site Selection versus Site Search

Site Selection

Site Search

study area, subdivided into a set of basic units of analysis (e.g. polygons, rasters)

classification of the units according to their suitability for a particular activity

aggregating the basic units of observations to determine spatial characteristics of the site such as its shape, contiguity, and/or compactness